

## Three flavor implications of the result of the CHOOZ Collaboration

Mohan Narayan and G. Rajasekaran  
*Institute of Mathematical Sciences, Chennai 600 113, India*

S. Uma Sankar

*Department of Physics, Indian Institute of Technology, Powai, Mumbai 400076, India*

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We analyze the recent result of the CHOOZ Collaboration in the context of mixing and oscillations between all the three neutrino flavors. If one assumes the hierarchy among the vacuum mass eigenvalues  $\delta_{21} \ll \delta_{31}$  where  $\delta_{21} = \mu_2^2 - \mu_1^2$  and  $\delta_{31} = \mu_3^2 - \mu_1^2$ , then the CHOOZ result puts a strong constraint on the allowed values of the (13) mixing angle  $\phi$ . It is also shown that, in light of the CHOOZ result, the maximum contribution of the  $\nu_\mu \leftrightarrow \nu_e$  oscillation channel to the atmospheric neutrino anomaly is less than 7 percent, thus demonstrating that the atmospheric neutrino anomaly is mainly due to  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations. Most importantly the CHOOZ result now excludes a large part of the three flavor parameter space which was previously allowed as solutions to the solar and atmospheric neutrino problems.

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The CHOOZ Collaboration, which searches for signals of  $\bar{\nu}_e \rightarrow \bar{\nu}_x$  oscillations, where  $x$  can be any other flavor, in the disappearance mode of the original flavor has recently reported the results of its first run [1]. They see no evidence of oscillations of the original flavor. They have analyzed their results assuming two flavor oscillations between  $\nu_e$  and another flavor and gave an exclusion plot in the parameter space spanned by the mass squared difference  $\Delta m^2$  and the mixing angle  $\theta$ . Their main result is that for  $\Delta m^2 > 3 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2(2\theta)$  must be less than 0.18. While this is a strong constraint, we remark that it has to be confirmed by an independent experiment. Nevertheless we may ask what are the consequences if we accept the CHOOZ result.

We reinterpret the CHOOZ result in terms of oscillations between the three active neutrino flavors [2–5]. This is a more realistic framework because it is established that there are three light neutrino flavors whose interactions are prescribed by the standard model. It is more natural to assume that all three of the light neutrinos mix with one another.

The flavor eigenstates are related to the mass eigenstates by

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}. \quad (1)$$

Here we can take, without loss of generality, that  $m_3 > m_2 > m_1$ . The unitary matrix  $U$  can be parametrized as [2]

$$U = U^{23}(\psi) \times U^{phase} \times U^{13}(\phi) \times U^{12}(\omega), \quad (2)$$

where  $U^{ij}(\theta_{ij})$  is the two flavor mixing matrix between the  $i$ th and  $j$ th mass eigenstates with the mixing angle  $\theta_{ij}$ . For simplicity, we neglect the  $CP$  violation and set  $U^{phase} = I$ . Explicitly,  $U$  can be written as

$$U = \begin{bmatrix} c_\phi c_\omega & c_\phi s_\omega & s_\phi \\ -c_\psi s_\omega - s_\psi s_\phi c_\omega & c_\psi c_\omega - s_\psi s_\phi s_\omega & s_\psi c_\phi \\ s_\psi s_\omega - c_\psi s_\phi c_\omega & -s_\psi c_\omega - c_\psi s_\phi s_\omega & c_\psi c_\phi \end{bmatrix} \quad (3)$$

where  $s$  and  $c$  stand for sine and cosine respectively.

The vacuum oscillation probability for a neutrino of flavor  $\alpha$  to oscillate into a neutrino of flavor  $\beta$  is given by

$$\begin{aligned} P_{\alpha\beta} = & (U_{\alpha 1} U_{\beta 1})^2 + (U_{\alpha 2} U_{\beta 2})^2 + (U_{\alpha 3} U_{\beta 3})^2 \\ & + 2 U_{\alpha 1} U_{\alpha 2} U_{\beta 1} U_{\beta 2} \cos\left(2.53 \frac{d \delta_{21}}{E}\right) \\ & + 2 U_{\alpha 1} U_{\alpha 3} U_{\beta 1} U_{\beta 3} \cos\left(2.53 \frac{d \delta_{31}}{E}\right) \\ & + 2 U_{\alpha 2} U_{\alpha 3} U_{\beta 2} U_{\beta 3} \cos\left(2.53 \frac{d \delta_{32}}{E}\right), \end{aligned} \quad (4)$$

where  $d$  is the distance traveled in meters,  $E$  is in MeV, and mass squared differences  $\delta_{ij} (= \mu_i^2 - \mu_j^2)$  are in  $\text{eV}^2$ . We may also note that the vacuum oscillation probabilities are the same as in Eq. (4) for the case of antineutrinos because  $CP$  violation is neglected. If we assume the hierarchy among the neutrino mass eigenstates  $\delta_{21} \ll \delta_{31} \simeq \delta_{32}$ , and that  $\delta_{21}$  is about  $10^{-5} \text{ eV}^2$ , which is required to fit solar neutrino data [6], then the oscillatory term involving  $\delta_{21}$  can be set to one. In this case the probability in Eq. (4) reduces to

$$\begin{aligned} P_{\alpha\beta} = & (U_{\alpha 1} U_{\beta 1} + U_{\alpha 2} U_{\beta 2})^2 + (U_{\alpha 3} U_{\beta 3})^2 \\ & + 2(U_{\alpha 1} U_{\beta 1} + U_{\alpha 2} U_{\beta 2}) U_{\alpha 3} U_{\beta 3} \cos\left(2.53 \frac{d \delta_{31}}{E}\right). \end{aligned} \quad (5)$$

The oscillation probability relevant for the CHOOZ experiment is the electron neutrino survival probability  $P_{ee}$ . Substituting  $\alpha=e=\beta$  in Eq. (5) and using the unitarity of  $U$ , we get

$$P_{ee} = (1 - U_{e3}^2)^2 + U_{e3}^4 + 2(1 - U_{e3}^2)U_{e3}^2 \cos\left(2.53 \frac{d\delta_{31}}{E}\right). \quad (6)$$

From Eq. (3), we see that  $U_{e3} = \sin \phi$ . Hence the electron neutrino survival probability is

$$P_{ee} = 1 - \sin^2 2\phi \sin^2\left(1.27 \frac{d\delta_{31}}{E}\right). \quad (7)$$

Notice the interesting point that this involves only the (13) mixing angle  $\phi$ , and because of the hierarchy the (12) mixing angle  $\omega$  disappears from the probability. So we reinterpret the CHOOZ result [1], to be that for  $\delta_{31} > 3 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2(2\phi)$  must be less than 0.18, i.e.  $\phi < 12.5^\circ$ .

We now estimate the maximum contribution of the  $e - \mu$  channel to the atmospheric neutrino anomaly. Since the relevant  $\delta_{31}$  is about  $10^{-2} \text{ eV}^2$ , matter effects are negligible for the problem [3]. Hence the relevant probability is the vacuum  $\nu_e \leftrightarrow \nu_\mu$  oscillation probability,

$$P_{\mu e} = P_{\mu e} = \sin^2 2\phi \sin^2 \psi \sin^2\left(1.27 \frac{d\delta_{31}}{E}\right). \quad (8)$$

Note that both  $\phi$  and  $\psi$  have to be nonzero for  $P_{\mu e}$  to be nonzero, and also the oscillation length corresponding to  $\delta_{21}$  does not contribute to the atmospheric neutrino problem [3]. Now solutions to Kamiokande atmospheric neutrino data [3,4] require a value of  $30^\circ \leq \psi \leq 60^\circ$  for small values of  $\phi$ . The average contribution of the oscillatory term is 0.5. Therefore the CHOOZ result implies that

$$P_{\mu e}^{\max} \leq 0.18 \times 0.75 \times 0.5 = 0.07 \quad (9)$$

which is less than 7 percent. Hence the atmospheric neutrino anomaly is driven almost completely by  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations. The  $\nu_e \leftrightarrow \nu_\tau$  conversion probability is given by

$$P_{e\tau} = P_{e\tau} = \sin^2 2\phi \cos^2 \psi \sin^2\left(1.27 \frac{d\delta_{31}}{E}\right). \quad (10)$$

The allowed range of  $\psi$  for small values of  $\phi$  once again gives an upper limit of 0.07 for the  $e - \tau$  conversion probability, i.e. the electron neutrino flux is hardly converted to other flavors, which is what is experimentally observed.

Lastly we incorporate the CHOOZ constraints on our previous fits to solar and atmospheric neutrino data, and so we reproduce the plots from our earlier works, with the constraints coming from the CHOOZ results shown on them. In Fig. 1, the light contours enclose the parameter region in  $\phi - \psi$  plane allowed by the binned multi-GeV data of Kamiokande with  $1.6\sigma$  error bars. The present CHOOZ constraint has been shown as a thick vertical line, with the region to the right of it being excluded. Figure 2 shows the allowed region in the  $\phi - \delta_{31}$  plane from the same analysis, with the

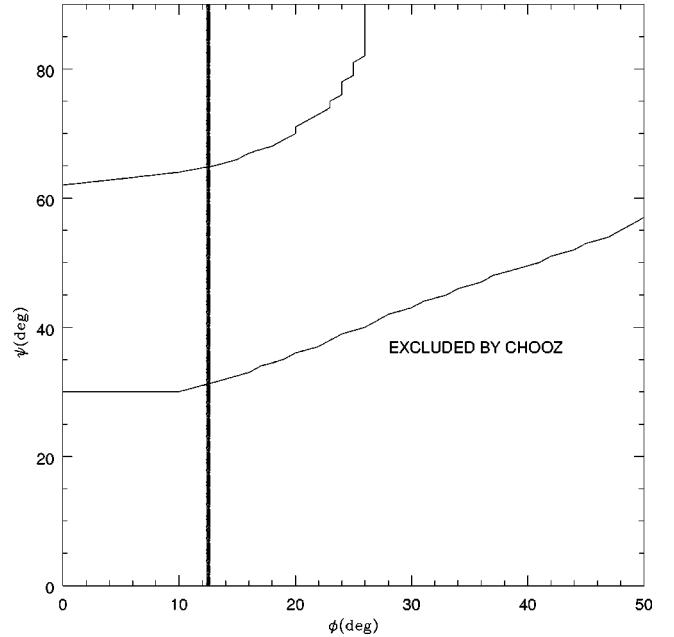


FIG. 1. Allowed parameter region in  $\phi - \psi$  plane by Kamiokande binned multi-GeV data with  $1.6\sigma$  error bars (light lines) and the new constraint by CHOOZ (thick line).

CHOOZ constraint again being shown as a thick vertical line [3]. Figures 3 and 4 show the previously allowed regions by the solar neutrino data in  $\phi - \omega$  and  $\phi - \delta_{21}$  planes respectively along with the new constraint [5].

In Ref. [3] the results of the Liquid Scintillation Neutrino Detector (LSND) Collaboration [7] were analyzed in the same three flavor framework, along with the atmospheric neutrino problem. It was found that there is a small region of overlap between the respective parameter spaces allowed by

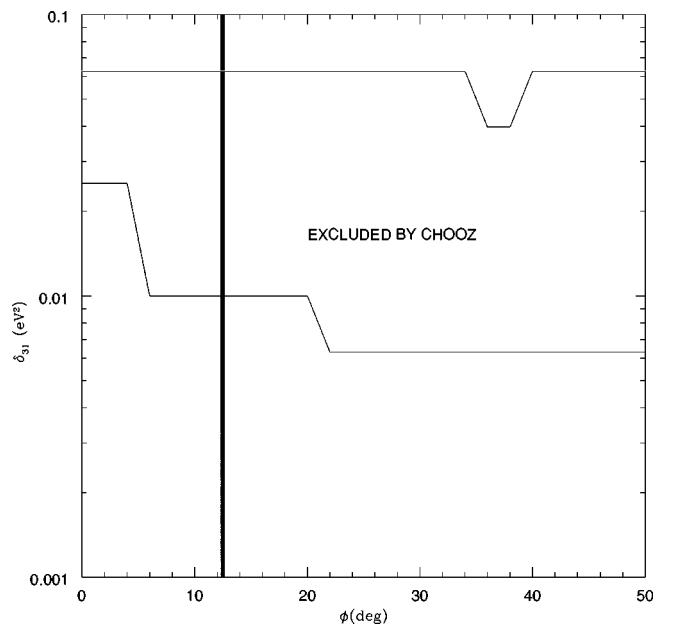


FIG. 2. Allowed parameter region in  $\phi - \delta_{31}$  plane by Kamiokande binned multi-GeV data with  $1.6\sigma$  error bars (light lines) and the new constraint by CHOOZ (thick line).

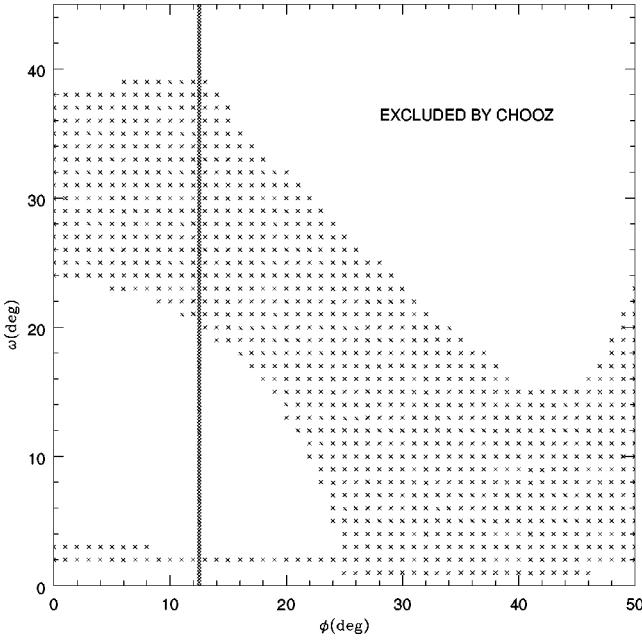


FIG. 3. Allowed parameter region in  $\phi - \omega$  plane by solar neutrino data with  $1.6\sigma$  error bars (crosses) and the new constraint by CHOOZ (thick line).

each experiment. Hence one could account for the solar neutrino problem, the atmospheric neutrino problem and the LSND results in a three flavor framework. This is no longer possible if one takes the CHOOZ result into account. If the CHOOZ constraint  $\phi \leq 12.5^\circ$  is imposed, then the lower limit on  $\delta_{31}$  from LSND goes up to about  $0.1 \text{ eV}^2$ , which is larger than the maximum allowed value from the atmospheric neutrino analysis. Hence it is not possible to explain the solar and the atmospheric neutrino problems and satisfy the results of the CHOOZ and LSND experiments in a three neutrino flavor framework.

Note the fact that  $\phi$  being the angle which connects the solar neutrino parameter space spanned by  $\omega$ ,  $\phi$ , and  $\delta_{21}$  with the atmospheric neutrino space spanned by  $\phi$ ,  $\psi$ , and  $\delta_{31}$ , the constraint on  $\phi$  also translates into a strong constraint in the solar neutrino parameter space [8,5]. Now ob-

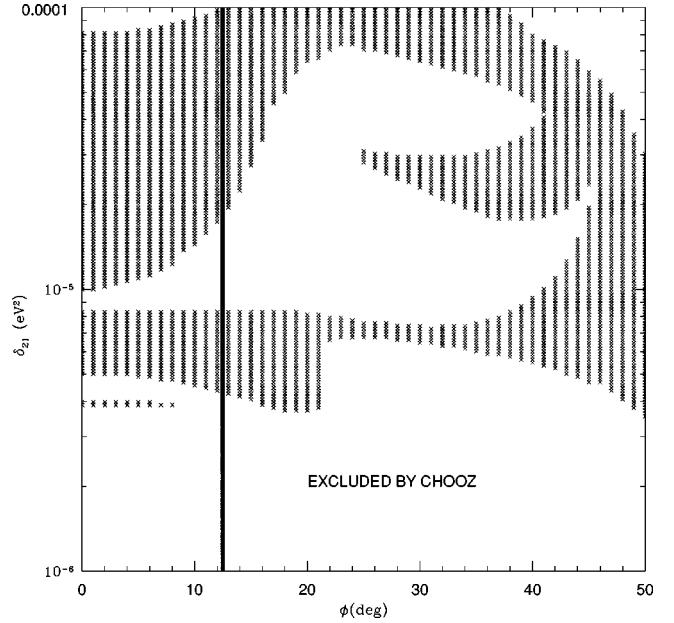


FIG. 4. Allowed parameter region in  $\phi - \delta_{21}$  plane by solar neutrino data with  $1.6\sigma$  error bars (crosses) and the new constraint by CHOOZ (thick line).

serve what is probably the most important consequence of the CHOOZ result. The fact that  $\phi$ , the link between the solar and the atmospheric neutrino problems, is constrained to be small implies that *the solar neutrino problem can be essentially viewed as a two flavor  $\nu_e \leftrightarrow \nu_\mu$  oscillation phenomenon, and the atmospheric neutrino problem essentially as a two flavor  $\nu_\mu \leftrightarrow \nu_\tau$  oscillation phenomenon even in a three flavor framework*.

In conclusion the recent CHOOZ result limits the  $\nu_\mu \leftrightarrow \nu_e$  contribution to the atmospheric neutrino anomaly as a function of the (13) mixing angle  $\phi$ , establishes the fact that the atmospheric neutrino anomaly is mainly  $\nu_\mu \leftrightarrow \nu_\tau$ , i.e. vacuum oscillations, and excludes large parts of the parameter space previously allowed as solutions to solar and atmospheric neutrino data.

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- [1] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998).
- [2] T. K. Kuo and J. Pantaleone, Phys. Rev. D **35**, 3432 (1987).
- [3] M. Narayan, G. Rajasekaran, and S. Uma Sankar, Phys. Rev. D **56**, 437 (1997).
- [4] G. L. Fogli, E. Lisi, D. Montanino, and G. Scosca, Phys. Rev. D **55**, 4385 (1997).
- [5] M. Narayan, M. V. N. Murthy, G. Rajasekaran, and S. Uma Sankar, Phys. Rev. D **53**, 2809 (1996).
- [6] N. Hata and P. Langacker, Phys. Rev. D **56**, 6107 (1997).
- [7] LSND Collaboration, C. Athanassopoulos *et al.*, Phys. Rev. Lett. **77**, 3082 (1996).
- [8] J. Pantaleone, Phys. Rev. D **49**, R2152 (1994).